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THE INVESTIGATION OF THE STABILITY OF ELASTIC AND VISCOELASTIC RODS UNDER A STOCHASTIC EXCITATION

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Abstract—The stability of elastic and viscoelastic rods subjected to a random stationary longitudinal force is considered. A numerical method of simulation of random realizations of a wide-band stationary process, corresponding to the variation of the longitudinal force in time, is employed. For each realization the numerical solution of the system of differential or integro-differential equations describing the dynamic behavior of the rods is found. With the help of Liapunov exponents which are calculated for statistical moments of the solution of the equations, the conclusion is made about the stability with respect to statistical moments. The described approach is applicable for the solution of similar stochastic problem for plates and shells under the linearized treatment.

1. INTRODUCTION

Problems of the stability of elastic and viscoelastic rods subjected to the longitudinal forces, which are stochastic stationary processes, were considered by many authors. The greatest number of results were obtained in the case when the stationary process is supposed to be a Gaussian white noise. First of all, it must be noted that sufficient conditions for the almost sure stability of the solution of differential equations under a random perturbation of their parameters were obtained by Kushner (1967) and Khasminskii (1980). For a special case of the differential equation of the second order, which describes the behavior of elastic rods under the stochastic treatment of the problem, similar results were found by Bolotin (1979) and Kozin (1986). Analogous stability conditions for viscoelastic rods were obtained by Tylikowski (1990) and Potapov (1993b). Kozin and Prodromou (1971) found sufficient and necessary conditions of the almost sure stability of linear Ito equations.

A great number of works were devoted to obtaining the stability boundaries with respect to statistical moments of the solution of differential equations (Khasminskii, 1980; Bolotin, 1979; Kozin, 1986; Potapov, 1985, 1992, 1994a; Drozdov and Kolmanovskii, 1992).

If a longitudinal force is a stationary wide-band process then the problem solution becomes significantly more complicated. Exact results in the sense of stability with respect to statistical moments of different order for the elastic rod, lying in a viscous medium, were found by Potapov (1989). The longitudinal force was assumed in the form of a stationary process with a fractional-rational spectral density. Sufficient conditions for the almost sure stability for elastic systems were obtained by Infante (1968), Wiens and Sinha (1984), Ahmadi Goodarz (1977) and Ariaratnam and Ly (1989). Similar results for viscoelastic rods were found by Potapov (1994b, c).

It should be stressed that the estimations of stability boundaries, which are obtained with the help of these sufficient conditions, usually are rather rough. More exact results can be obtained using an asymptotic method, which is also known as a method of stochastic averaging (Ariaratnam, 1967, 1972; Dimentberg, 1980, 1989; Potapov, 1984, 1985). But these results are applicable only in those cases when the measurements of the material viscosity and/or external damping are small enough and random fluctuations of the longitudinal force have a small mean-square scattering. This method was proposed by Stratonovich (1963) and proved rigorously by Khasminskii (1966) in a limit theorem.

If the indicated restrictions are invalid then other methods should be applied for the stability investigation. One of them is a numerical method of statistical simulation (Potapov, 1990; Potapov and Marasanov, 1992). With the help of this method the problem of the stability can be solved fairly easily for a finite time interval, and is connected with the first passage problem in the theory of random processes.

In the past time Liapunov exponents have found a wide application for the solution of the stability problem in different fields of science (Benettin *et al.*, 1980). In particular, the dynamic stability analysis of viscoelastic structures by Liapunov exponents was carried out in the works of Aboudi *et al.* (1990) and Cederbaum *et al.* (1991). The asymptotic method together with Liapunov exponents was used for the investigation of almost sure stability of stochastic systems by Ariaratnam and Ly (1989), Ariaratnam *et al.* (1990) and Ariaratnam and Wei-Chau Xie (1992). The present work is devoted to the investigation of the stability of elastic and viscoelastic rods on the basis of the method of statistical simulation combined with the Liapunov exponents method in assumption that the kernel of the material relaxation is represented by the sum of exponents.

2. TREATMENT OF THE PROBLEM

The dynamic behavior of a flexible perfect viscoelastic rod at small deflections is described by the equation

$$m\frac{\partial^2 w}{\partial t^2}+k\frac{\partial w}{\partial t}+EI(1-\mathbf{R})\frac{\partial^4 w}{\partial x^4}+F\frac{\partial^2 w}{\partial x^2}=0. \tag{1}$$

Here m is the mass per unit length, w is the transverse deflection, $k\partial w/\partial t$ is the term, describing the external damping, EI is the bending stiffness, F denotes the longitudinal force and $\mathbf{R}$ is the relaxation operator

$$\mathbf{R}\frac{\partial^4 w}{\partial x^4}\equiv\int_0^t R(t-\tau)\frac{\partial^4 w(\tau,x)}{\partial x^4}\mathrm{d}\tau \quad 0\leqslant\int_0^\infty R(\theta)\,\mathrm{d}\theta<1.$$

The function $w(t,x)$ should satisfy the corresponding initial and boundary conditions. Let us consider a simply supported rod with initial conditions

$$w(0,x)=f_0\sin\frac{\pi}{l}x,\quad \dot{w}(0,x)=v_0\sin\frac{\pi}{l}x.$$

Then the solution of equation (1) is written in the form

$$w(t,x)=f(t)\sin\frac{\pi}{l}x.$$

The deflection amplitude of the rod is found from the integro-differential equation

$$\ddot{f}+2\varepsilon^*\dot{f}+\omega^2[(1-\mathbf{R})-\alpha]f=0 \tag{2}$$

where

$$2\varepsilon^*=\frac{k}{m}\quad \omega^2=\frac{\pi^4 EI}{ml^4}\quad \alpha=\frac{Fl^2}{\pi^2 EI}.$$

The dot denotes differentiation with respect to the time t.

Further let us assume that the kernel of the material relaxation $R(t-\tau t)$ is represented by the sum of exponents

$$R(t-\tau) = \sum_{i=1}^{k} \chi_i^* L_i \exp\left[-\chi_i^*(t-\tau)\right].$$

After the introduction of the new variables (Tylikowski, 1990; Potapov and Marasanov, 1992)

$$x_1 = f, \quad x_2 = \dot{f}, \quad x_{2+i} = \int_0^t \chi_i^* L_i \exp\left[-\chi_i^*(t-\tau) f(\tau)\right] \mathrm{d}\tau, \quad i = 1, 2, \ldots, k$$

equation (2) may be replaced by the following system of differential equations of the first order. If we use the dimensionless time $t_1 = \omega t$, then this system is written in the form

$$\mathbf{x}' = \mathbf{A}\mathbf{x} \tag{3}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ -(1-\alpha) & -2\varepsilon & 1 & \ldots & 1 \\ \chi_1 L_1 & 0 & -\chi_1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \chi_k L_k & 0 & 0 & \ldots & -\chi_k \end{bmatrix} \quad \varepsilon = \varepsilon^*/\omega \quad \chi_i = \chi_i^*/\omega.$$

The prime denotes differentiation with respect to dimensionless time t_1. Furthermore, for the sake of convenience we will again use the notation t instead of t_1.

The same approach can be used in the case when the creep curve can be described by the expression (Arutunian, 1952; Potapov, 1993a)

$$\varepsilon(t) = \left[\frac{1}{E(\tau)} + C(t,\tau)\right]\sigma_0$$

$$C(t,\tau) = \xi(\tau) \sum_{k=0}^{n} B_k \exp\left[-\gamma_k(t-\tau)\right]$$

if the functions $E(\tau)$, $\xi(\tau)$ approach constant values, for example, if they are written in the forms

$$E(\tau) = E_0(1-\mathrm{e}^{-\beta\tau}),$$

$$\xi(\tau) = C_0 + \sum_{i=1}^{m} A_i \exp(-\beta_i \tau)$$

or

$$\xi(\tau) = C_0 + \sum_{i=1}^{m} A_i/(\tau_i + \tau)$$

Here $\beta > 0$, $\beta_i > 0$, $A_i > 0$, $\theta_i > 0$, $C_0 > 0$, $B_k \geqslant 0$.

In this case the integro-differential equation (2) can be written as a system of differential equations of the first order with varying coefficients approaching, with the passage of time, to the constant values (Arutunian, 1952; Potapov, 1993a).

Let us presuppose that the function $\alpha(t)$ can be written in the form

$$\alpha(t) = \alpha_0 + \varphi(t)$$

where $\alpha_0 = \text{const}$, $\varphi(t)$ is a wide-band stationary process, the mean value of which is equal to zero.

3. STABILITY OF THE TRIVIAL SOLUTION

The principal purpose of the present work is the investigation of the stability of the trivial solution of system (3). Nowadays different formulations of the stability of stochastic systems are well-known. We shall recall some of them, which are used later.

(i) The solution $\mathbf{x}(t) \equiv 0$ is called p-stable, if for any $\varepsilon > 0$, such $\delta > 0$ can be found that at $t \geqslant 0$ and $|x_i(0)| < \delta$

$$|\langle x_i^p(t) \rangle| < \varepsilon.$$

Angle brackets denote the expected value (ensemble average).

(ii) The solution $\mathbf{x}(t) \equiv 0$ is called asymptotically p-stable, if it is p-stable and, in addition, for sufficiently small $|x_i(0)|$

$$\lim_{t\to\infty} |\langle x_i^p(t) \rangle| = 0.$$

At $p = 1$, stability in the mean takes place while at $p = 2$, stability in the mean-square occurs.

For the numerical solution of the stability problem we will take advantage of the method of canonical expansion of stationary random process $\varphi(t)$. It is well known that any stationary process $\varphi(t)$ for a finite time interval $[0, T]$ can be represented by the series (Karhunen, 1947; Gikhman and Skorokhod, 1965)

$$\varphi(t) = \sum_{j=0}^{\infty} (U_j \cos\theta_j t + V_j \sin\theta_j t)$$

where U_j, V_j are uncorrelated random values, $\langle U_j \rangle = \langle V_j \rangle = 0$, $\langle U_j^2 \rangle = \langle V_j^2 \rangle$, $\theta_j = j\Delta\theta$, $\Delta\theta = 2\pi/T$.

For the numerical simulation of the process $\varphi(t)$ we can employ instead of series, the finite sum with a large value of N

$$\varphi(t) = \sum_{j=0}^{N} (U_j \cos\theta_j t + V_j \sin\theta_j t). \tag{4}$$

Here $\theta_j = j\Delta\theta$, $\Delta\theta = \theta_u/N$, $\langle U_0^2 \rangle = S(0)\Delta\theta$, $\langle U_j^2 \rangle = \langle V_j^2 \rangle = 2S(\theta_j)\Delta\theta$, $j = 1, 2, \ldots, N$.

The value θ_u represents an upper cut-off frequency for the two-sided power spectral density function in such a way that beyond θ_u its value may be taken to be zero. Using realizations of the function $\varphi(t)$, obtained in this way, we can find, with the help of the Runge-Kutta method, the solution of system of equations (3).

Let us consider peculiarities of obtaining of Liapunov exponents for the investigation of the stability with respect to statistical moments. It is known that the value, defined by the expression

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \ln \frac{\|\mathbf{x}(t)\|}{\|\mathbf{x}(0)\|}$$

is the greatest Liapunov exponent, where $\|\mathbf{x}(t)\|$ is the norm of the vector $\mathbf{x}(t)$ in Euclidean space and $\|\mathbf{x}(0)\|$ is the norm of the vector of initial data. Numerically this value can be found by the method recommended by Benettin *et al.* (1980). With this purpose we will divide the enough large interval of time $[0, T]$ into n equal portions of length $\Delta = t_{i+1} - t_i$.

Let us suppose that at $t = t_i$ the norm $\|\mathbf{x}(t_i)\|$ is equal to unity. Using this vector as the vector of initial data we will find the solution of the system (3) for time t_{i+1} with the norm $\|\mathbf{x}(t_{i+1})\| = d_{i-1}$. By keeping on solving system (3) with new initial data $x_j(t_{i+1})/d_{i+1}$ $(j = 1, 2, \ldots, 2+k)$ we will obtain the sequence of values d_i and the greatest Liapunov exponent can be found as the limit

$$\lambda_{\max} = \lim_{n\to\infty} \frac{1}{n\Delta} \sum_{i=1}^{n} \ln d_i. \tag{5}$$

If the stability with respect to statistical moments is considered, then the solution is formed in the following way. Since the closed system of equations for the moments of the function $x_i(t)$ in the case of wide-band stationary process could not be obtained, we will use the method of statistical data processing. Let us explain this approach for the example of moments of the second order and stability in the mean-square. The system of equations with respect to all kinds of products $x_i x_j$ $(i, j = 1, 2, \ldots, 2+k)$ can be obtained from the system of equations (3). The number of such equations is equal to $m = (2+k)(3+k)/2$.)

$$\dot{\mathbf{z}} = \mathbf{B}\mathbf{z} \tag{6}$$

where

$$\mathbf{z}^{\mathrm{T}} = [z_1 \quad z_2 \quad \ldots \quad z_m] \equiv [(x_1)^2 \quad (x_1 x_2) \quad \ldots \quad (x_1 x_{2+k}) \quad \ldots \quad (x_{2+k}^2)].$$

In particular for $k = 1$ the matrix **B** and the vector **z** are written as

$$\mathbf{B} = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ -(1-\alpha) & -2\varepsilon & 1 & 1 & 0 & 0 \\ \chi L & 0 & -\chi & 0 & 1 & 0 \\ 0 & -2(1-\alpha) & 0 & -4\varepsilon & 2 & 0 \\ 0 & \chi L & -(1-\alpha) & 0 & -(2\varepsilon+\chi) & 1 \\ 0 & 0 & 2\chi L & 0 & 0 & -2\chi \end{bmatrix}$$

$$\mathbf{z}^{\mathrm{T}} = [z_1 \quad z_2 \quad \ldots \quad z_6] \equiv [(x_1^2) \quad (x_1 x_2) \quad (x_1 x_3) \quad (x_2^2) \quad (x_2 x_3) \quad (x_3^2)].$$

Further we can solve this system for each realization of the process $\alpha(t)$ and find estimations of mathematical expectations of components of the vector **z**

$$\langle z_j \rangle = \frac{1}{q} \sum_{i=1}^{q} z_j^{(i)}, \quad j = 1, 2, \ldots, 6 \tag{7}$$

where q is the number of realizations, $z_j^{(i)}$ is the magnitude of z_j in ith realization.

Let us suppose that at some moment of time the expression

$$\tilde{D}_n = \left[\sum_{j=1}^{m} \langle z_j(t_n) \rangle^2 \right]^{1/2}$$

is equal to unity. It should be stressed that with the increase of the number q estimation (7) approaches the mathematical expectation $\langle z_j(t_n) \rangle$ and the value $\tilde{D}_n(t_n)$, to the norm of the vector $\langle \mathbf{z}(t_n) \rangle$ in Euclidean space

$$D_n = \left[\sum_{j=1}^{m} \langle z_j(t_n) \rangle^2 \right]^{1/2}.$$

Assuming the values $z_j(t_n)$ as initial data for system (6) we will find its solution for the interval of time $[t_n, t_{n+1}] \equiv [t_n, t_n+\Delta]$. Let us obtain again the values $\langle \tilde{z}_j(t_{n+1}) \rangle$ and $\tilde{D}_{n+1}$. Further system (6) is solved for each realization with initial data

$$z_{j0}(t_{n+1}) = z_j(t_{n+1})/\tilde{D}_{n+1}$$

The greatest Liapunov exponent with respect to $\langle z_i \rangle$ is equal to

$$\Lambda_{\max} = \lim_{n\to\infty} \frac{1}{n\Delta} \sum_{i=1}^{n} \ln D_i$$

therefore

$$\tilde{\Lambda} = \lim_{n\to\infty} \frac{1}{n\Delta} \sum_{i=1}^{k} \ln \tilde{D}_i \tag{8}$$

is an approximate of the value $\Lambda_{\max}$.

If $\Lambda_{\max}$ is negative then the rod is asymptotically stable in mean-square. It is obvious that this approach can be simply generalized for the case of stability with respect to statistical moments of order higher than the second one.

Such an approach, connected with the definition of moments of all kinds of products of components x_i, is employed usually in the cases, when the closed system of equations with respect to those moments can be formed (for example, if the function $\varphi(t)$ is a Gaussian white noise). Since in the general case obtaining the indicated system of equations is impossible then the solution of the formulated problem can be found, starting from consideration of equation (3) directly.

The estimation of moments $\langle x_j^p \rangle$ for time moment t_n can be obtained as a result of statistical average of values x_j^p, derived from the solution of equation (3) for the enough large number of realizations

$$\langle \tilde{x}_j^p(t_n) \rangle = \frac{1}{q} \sum_{i=1}^{q} [x_j^p(t_n)]^{(i)}$$

where $[x_j^p(t_n)]^{(i)}$ is the magnitude $x_j^p(t_n)$, corresponding to ith realization of the solution of equation (3).

Let us assume that the norm of the vector $\langle x^p(t_n) \rangle$ in Euclidean space in time moment t_n is equal to unity. In time moment $t_{n+1} = t_n + \Delta$ the norm of the vector $\langle x^p(t) \rangle$ becomes equal to $\tilde{d}_{n+1}$. The further system (3) is solved for each realization with initial data

$$x_{j0}(t_{n+1}) = x_j(t_{n+1})/(\tilde{d}_{n+1})^{1/p}.$$

As a result of repeated employment of this procedure the sequence of numbers $\tilde{d}_i$ is used to estimate the Liapunov exponent, defined as

$$\tilde{\lambda} = \lim_{n\to\infty} \frac{1}{n\Delta} \sum_{i=1}^{n} \ln \tilde{d}_i.$$

The advantage of this approach is contained in the solution of the system of differential equations, the order of which is lesser in comparison with the order of the system (6).

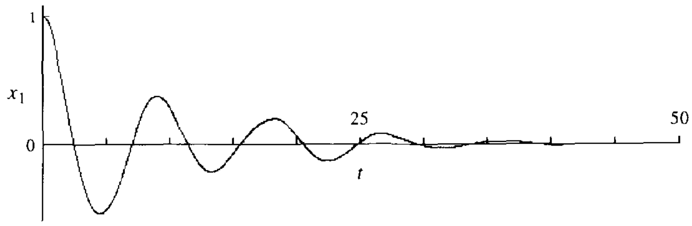

Fig. 1. The realization of the solution $x_1(t)$ for the elastic rod.

4. EXAMPLES

Example 1

Let us consider the elastic rod. Systems of eqns (3), (6) are written in the following way

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -(1-\alpha)x_1 - 2\varepsilon x_2 \end{aligned} \tag{9}$$

and

$$\dot{\mathbf{z}} = \mathbf{B}\mathbf{z} \tag{10}$$

where

$$\mathbf{B} = \begin{bmatrix} 0 & 2 & 0 \\ -(1-\alpha) & -2\varepsilon & 1 \\ 0 & -2(1-\alpha) & -4\varepsilon \end{bmatrix} \quad \mathbf{z} = \begin{bmatrix} x_1^2 \\ x_1x_2 \\ x_2^2 \end{bmatrix}.$$

The random function $\varphi(t)$ is assumed in the form of a Gaussian stationary process, with the correlation function

$$K(t) = \sigma^2 \exp(-\delta|t|). \tag{11}$$

One of the realizations of the solution $x_1(t)$, obtained for $\varepsilon = 0.1$, $\alpha_0 = 0.5$, $\sigma = 0.2$, $\delta = 0.5$, $\Delta t = 0.025$, $\theta_u = 50$, $N = 1000$ is represented in Fig. 1. The values $x_{1\max}$ and $x_{1\min}$ in this figure are equal to 1.000 and -0.563 for the input data $x_1(0) = 1.000$, $x_2(0) = 0$. A chart $\tilde{\Lambda}_{\max} \sim n\Delta$ at $q = 5$ is shown in Fig. 2.

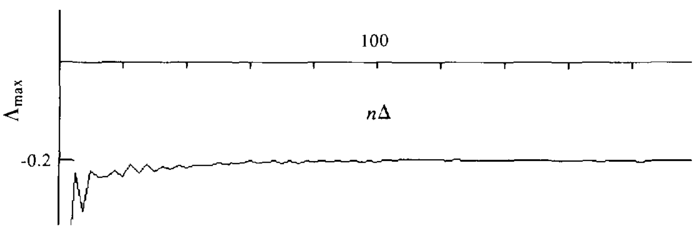

Fig. 2. The function $\tilde{\Lambda}_{\max} \sim n\Delta$.

Table 1. Liapunov exponents $\tilde{\Lambda}_{max}$ for the elastic rod as the function of the number of realizations q and mean-square scattering of the force ($N = 1000$)

q	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$
2	-0.200	-0.198	-0.184
5	-0.200	-0.197	-0.184
10	-0.200	-0.197	-0.183

Results of the solution of system (10) with the help of the Runge–Kutta method (of the fourth order) at $\alpha_0 = 0.5$; $\delta = 0.5$; $\varepsilon = 0.1$ are presented in Table 1. Table 1 illustrates the influence of the mean-square scattering of the longitudinal force (parameter σ) on the asymptotic stability in mean-square of the rod. It is interesting that the results, obtained with help of equations (9) and (10), coincide.

To test these results we will take advantage of the asymptotic method (Dimentberg, 1980) for the solution of the same problem. The solution of equations (9) is sought in the form

$$x_1(t) = A(t)\sin\Omega(t), \quad \dot{x}_2 = vA(t)\cos\Omega(t), \quad \Omega(t) = vt+\theta(t), \quad v^2 = 1-\alpha_0.$$

After some well known transformations we can find the following differential equation for the moment of second order of the amplitude $A(t)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle A^2\rangle = -2(\varepsilon-4\mu)\langle A^2\rangle \tag{12}$$

where

$$\mu = \frac{\pi}{8v^2}\Phi(2v), \quad \Phi(2v) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K(t)\cos(2vt)\,\mathrm{d}t.$$

In our case we have

$$\mu = \frac{1}{8}\frac{\sigma^2\delta}{(1-\alpha_0)[\delta^2+4(1-\alpha_0)]}.$$

The solution of equation (12) is

$$\langle A^2\rangle = C\exp\left[-2(\varepsilon-4\mu)t\right].$$

Thus, the Liapunov exponent Λ_{max} is equal to

$$\Lambda_{max} = -2(\varepsilon-4\mu). \tag{13}$$

At $\varepsilon = 0.1$; $\delta = 0.5$; $\alpha_0 = 0.5$; $\sigma = 0.1$ and $\sigma = 0.2$ from here it follows $\Lambda_{max} = -0.196$ and -0.182, respectively. The comparison of these values with corresponding magnitudes in Table 1 shows that the numerical and asymptotic methods give close enough results. We should bear in mind that expression (13) is obtained for a small mean-square scattering of the random function $\varphi(t)$ and a small value of the parameter ε. Therefore the value Λ_{max} is approximate too.

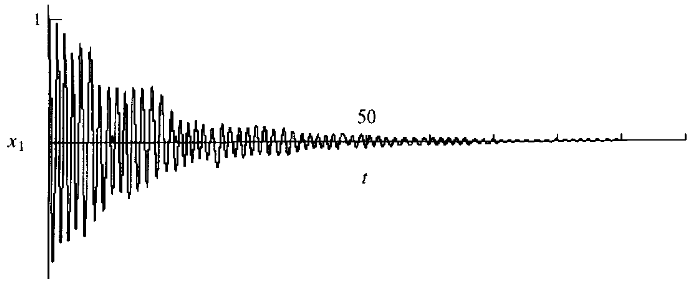

Fig. 3. The realization of the solution $x_1(t)$ for the viscoelastic rod.

Example 2

Let us consider the viscoelastic rod, the relaxation kernel of which has the form

$$R(t-\tau) = \chi L \exp[-\chi(t-\tau)].$$

The influence of viscous properties of the material on the behavior of the rod can be evaluated with help of realization $x_1(t)$, represented in Fig. 3. This chart corresponds to the input data $\varepsilon = 0, \chi = 0.1, L = 0.1, \alpha_0 = 0.5, \sigma = 0.2, \delta = 0.5, \Delta t = 0.025, \theta_u = 50, N = 1000$ and $x_1(0) = 1.000$, $x_2(0) = 0$, $x_3(0) = 0$. The values $x_{1\max}$ and $x_{1\min}$ in this figure are equal to 1.000 and -0.95.

In Table 2 the results are presented, which are found for the correlation function (11) and next input data $\varepsilon = 0$, $\chi = 0.1$, $L = 0.1$, $\alpha_0 = 0.5$, $\delta = 0.5$. For the test of these results the asymptotic method can also be applied (Potapov, 1984; 1985). The expression for the statistical moment of the second order for the deflection amplitude is written in our case as (at $\varepsilon = 0$)

$$\langle A^2 \rangle = C\exp(\lambda t),$$

where

$$\lambda = \frac{1}{1-\alpha_0}\left[\frac{\sigma^2\delta}{\delta^2+4(1-\alpha_0)} - \chi L\left(1+\frac{\chi^2}{1-\alpha_0}\right)^{-1}\right]. \tag{14}$$

At $\alpha_0 = 0.5$; $\chi = 0.1$; $L = 0.1$ $\sigma = 0.1$ and $\sigma = 0.2$ we have from here $\Lambda_{\max} = -0.015$ and -0.002, respectively.

The comparison of these values with corresponding magnitudes in Table 2 shows that the numerical and asymptotic method gives sufficiently close results for viscoelastic rods too. It must be noticed that the results, obtained with help of equation (3) at $\chi_1 = \chi$, $\chi_2 = \cdots = \chi_k = 0$, $L_1 = L$ and equation (6) coincide.

5. CONCLUSION

In the present work an effective numerical method for the investigation of the stability of elastic and viscoelastic rods under stochastic and periodic excitation is proposed. This

Table 2. Values of Liapunov exponents $\tilde{\Lambda}_{\max}$ for the viscoelastic rod as the function of the mean-square scattering of the force ($N = 1000, q = 10$)

σ	$\tilde{\Lambda}_{\max}$
0	-0.020
0.1	-0.018
0.2	-0.001

method is based on the calculation of Liapunov exponents. The comparison of the obtained results with analytical results, found with the help of the asymptotic method, shows that they are close enough. In the paper all arguments were made as applied columns, but the very same method can be used for the solution of the problem for perfect elastic and viscoelastic plates and shells under a linearized treatment. The considered method makes it possible to estimate the stability with respect to statistical moments of different order.

The suggested method can be used for the investigation of elastic and viscoelastic systems whose material properties and geometric parameters are random values or functions (Potapov, 1990; Potapov and Marasanov, 1992).

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